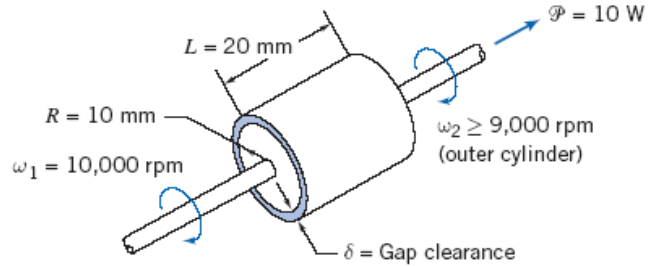


## Problem 2.64

[Difficulty: 3]

**2.64** A shock-free coupling for a low-power mechanical drive is to be made from a pair of concentric cylinders. The annular space between the cylinders is to be filled with oil. The drive must transmit power,  $\mathcal{P} = 10 \text{ W}$ . Other dimensions and properties are as shown. Neglect any bearing friction and end effects. Assume the minimum practical gap clearance  $\delta$  for the device is  $\delta = 0.25 \text{ mm}$ . Dow manufactures silicone fluids with viscosities as high as  $10^6$  centipoise. Determine the viscosity that should be specified to satisfy the requirement for this device.



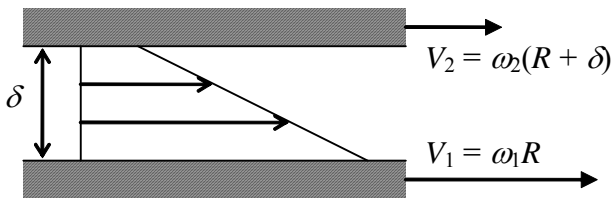
**Given:** Shock-free coupling assembly

**Find:** Required viscosity

**Solution:**

Basic equation  $\tau_{r\theta} = \mu \frac{du}{dr}$  Shear force  $F = \tau \cdot A$  Torque  $T = F \cdot R$  Power  $P = T \cdot \omega$

Assumptions: Newtonian fluid, linear velocity profile



$$\tau_{r\theta} = \mu \frac{du}{dr} = \mu \frac{\Delta V}{\Delta r} = \mu \frac{[\omega_1 \cdot R - \omega_2 \cdot (R + \delta)]}{\delta}$$

$$\tau_{r\theta} = \mu \frac{(\omega_1 - \omega_2) \cdot R}{\delta} \quad \text{Because } \delta \ll R$$

Then  $P = T \cdot \omega_2 = F \cdot R \cdot \omega_2 = \tau \cdot A_2 \cdot R \cdot \omega_2 = \frac{\mu (\omega_1 - \omega_2) \cdot R}{\delta} \cdot 2 \cdot \pi \cdot R \cdot L \cdot R \cdot \omega_2$

$$P = \frac{2 \cdot \pi \cdot \mu \cdot \omega_2 \cdot (\omega_1 - \omega_2) \cdot R^3 \cdot L}{\delta}$$

Hence  $\mu = \frac{P \cdot \delta}{2 \cdot \pi \cdot \omega_2 \cdot (\omega_1 - \omega_2) \cdot R^3 \cdot L}$

$$\mu = \frac{10 \cdot \text{W} \times 2.5 \times 10^{-4} \cdot \text{m}}{2 \cdot \pi} \times \frac{1}{9000} \cdot \frac{\text{min}}{\text{rev}} \times \frac{1}{1000} \cdot \frac{\text{min}}{\text{rev}} \times \frac{1}{(.01 \cdot \text{m})^3} \times \frac{1}{0.02 \cdot \text{m}} \times \frac{\text{N} \cdot \text{m}}{\text{s} \cdot \text{W}} \times \left( \frac{\text{rev}}{2 \cdot \pi \cdot \text{rad}} \right)^2 \times \left( \frac{60 \cdot \text{s}}{\text{min}} \right)^2$$

$$\mu = 0.202 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2} \quad \mu = 2.02 \cdot \text{poise} \quad \text{which corresponds to SAE 30 oil at } 30^\circ\text{C}.$$